The Generalized Minimum Spanning Tree (GMST) Problem

Problem Definition
Given:
• undirected complete Graph G=(V, E, c)
• node set V partitioned into disjoint clusters V₁, ..., Vₚ

Wanted:
A minimum cost spanning tree containing exactly one node from every cluster.

Selected Nodes Representation
• represent solution by used nodes p∈(V₁, ..., Vₚ)
• derive best spanning tree with Kruskals MST algorithm in O(n²).

Global Tree Representation
• represent solution by set of global edges (i.e. connections between clusters)
• compute best nodes with dynamic programming in O(n³).

Graph Reduction
• fix used nodes in all clusters with degree greater than two.
• determine the best nodes for all other clusters with dynamic programming.

Exact Methods
• Local - Global Multicommodity Flow based formulation
• Extended Node Optimization NB is able to compute best nodes using ILP
• only applicable for small to medium sized problem instances

Hybrid Variable Neighborhood Search Approach to solve the GMST Problem

Node Exchange Neighborhood
• change exactly one used node

Restricted Two Nodes Exchange Neighborhood
• selected nodes representation
• change used nodes of two adjacent clusters

Global Subtree Optimization Neighborhood
• global tree representation
• solve a small subproblem exact with ILP

Global Edge Exchange Neighborhood
• global tree representation
• exchange exactly one global edge by another one

Computational Results
proposed VNS algorithm has significant advantages over previous metaheuristic approaches (in particular on instances with a large number of nodes per cluster).

The Generalized Minimum Edge Biconnected Network (GMEBCN) Problem

Problem Definition
Given:
• undirected complete Graph G=(V, E, c)
• node set V partitioned into disjoint clusters V₁, ..., Vₚ

Wanted:
A minimum cost subgraph containing one node from each cluster that contains no bridges.

Selected Nodes Representation
If the used nodes are fixed, determining the best edges is NP hard.

Global Tree Representation
If the global edges between clusters are fixed, determining the best nodes is NP hard.

Graph Reduction

Variable Neighborhood Search Approaches to solve the GMEBCN Problem

Simple Node Optimization Neighborhood
• change exactly one used node
• global edges are retained

Node Re-Arrangement Neighborhood
• swap the sets of incident edges for two nodes

Cluster Re-Arrangement Neighborhood
• extends the Node Re-Arrangement Neighborhood
• operates on reduced graph

Extended Node Optimization Neighborhood
• extends the Simple Node Optimization Neighborhood
• operates on reduced graph

Node Exchange Neighborhood
• remove a single node and its incident edges
• select a new node
• augment the solution until it is edge biconnected again

Computational Results
• VNS₂ uses the more sophisticated neighborhoods based on graph reduction
• VNS₁ uses only the simpler, but faster neighborhoods
• ACH generates a feasible initial solution
• VNS₂ significantly outperforms VNS₁